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A NOTE ON THE APPLICATION OF THE AVERAGE CORRECTION TECHNIQUE

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SUMMARY

This note develops an average correction technique for accelerating the rate of convergence of the SIMPLE-like algorithm by implementing the average pressure correction method as proposed by Wen and Ingham (*Int. j. numer. methods fluids*, **17**, 385–400 (1993); **19**, 889–903 (1994)) with an average velocity correction. The technique is illustrated by considering the classical problem of fluid flow over a backward-facing step using (i) no average correction, (ii) an average velocity correction, (iii) an average pressure correction and (iv) both average velocity and pressure corrections. When both average velocity and pressure corrections are employed, it is found that the number of iterations required for convergence is almost independent of the initial guessed values of fluid velocity and pressure and the fastest rate of convergence may be achieved.

KEY WORDS: SIMPLE-like algorithms; average correction technique

1. INTRODUCTION

For cases where there is a rapidly varying pressure in a fluid flow problem, an average pressure correction technique has been developed by Wen and Ingham[1,2] in order to improve the rate of convergence of the SIMPLE-like algorithm when solving both the laminar and turbulent Navier–Stokes equations. In those papers only the average pressure correction was used on the one or two lines in the solution domain where there was a rapid variation in pressure and the technique led to a significant improvement in the rate of convergence of the SIMPLE-like algorithm. However, there is a lack of information on how this technique is able to deal with more general situations. In this note we illustrate that if only the average pressure correction is used to implement the SIMPLE-like algorithm, then the procedure diverges when the fluid velocity, either the guessed initial value or the updated value of the SIMPLE-like algorithm, is in the direction opposite to that of the average solution velocity. Also, we will show that if only the average velocity correction is employed in the implementation of the SIMPLE- like algorithm, then the rate of convergence is slower than for the SIMPLE- like algorithm without the average velocity correction. In this note we describe (i) how to

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use both average pressure and velocity corrections in a part or the whole of the solution domain, (ii) how the average pressure and velocity corrections affect the rate of convergence and (iii) how an average correction technique can be developed to obtain a better and faster rate of convergence.

2. THE AVERAGE CORRECTION TECHNIQUE

For two- or three-dimensional fluid flows the average correction should be applied along a certain direction. For example, Figure 1(a) shows the distribution of the updated average velocity component in the x- direction, U_i^*, and the updated average pressure along the x-direction, P_i^*. Here U_i^* and P_i^* are produced by the SIMPLE-like algorithm and, for convenience, P_i^* denotes the average pressure which is located downstream but adjacent to the line U_i^*. If $u_i^*(y)$ is the distribution of the x-component of the fluid velocity along the line i, calculated by the SIMPLE-like algorithm, then U_i^* is given by

$$U_i^* = \frac{1}{A}\int_A u_i^*(y)\,\mathrm{d}s, \qquad (1)$$

where A is the cross-sectional area of the line i.

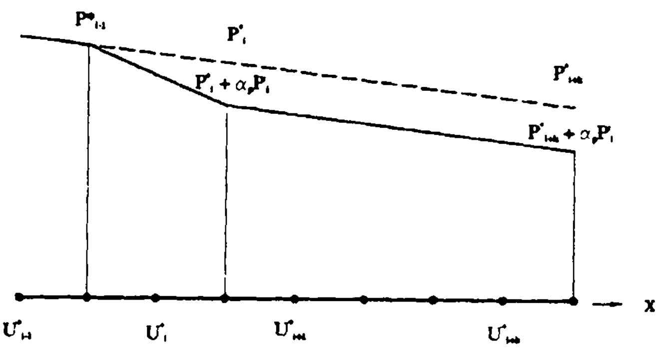

Figure 1(a). Average corrected pressure distribution along x-direction when average correction is only applied on one line i

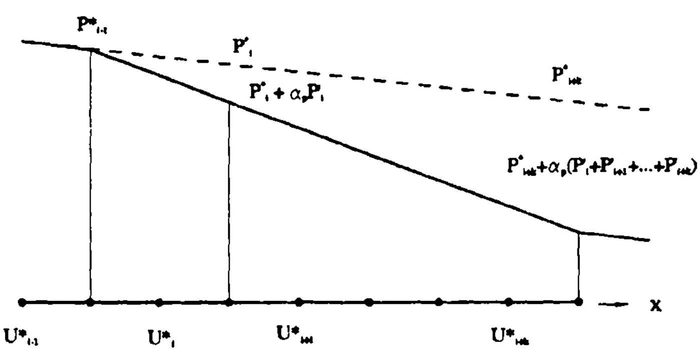

Figure 1(b). Average corrected pressure distribution along x-direction when average correction is applied from line i to line $i+k$

If the flux of fluid in the x-direction is Q, then on the line i the average velocity correction U'_i is obtained by using the global mass conservation principle, namely

$$U'_i = U_i - U^*_i, \tag{2}$$

where $U_i = Q/A$.

If we let P'_i be the average pressure correction, then P'_i is calculated using the formula[1]

$$P'_i = -\rho(U^*_i U'_i + U'^2_i/2). \tag{3}$$

Omitting the small quantity $U'^2_i/2$ from equation (3) yields

$$P'_i = -\rho U^*_i U'_i. \tag{4}$$

Further simplifications can be made by replacing the updated average velocity U^*_i by the real average velocity U_i; then equation (4) reads

$$P'_i = -\rho U_i U'_i. \tag{5}$$

If we apply the average velocity and pressure corrections on only one line, say the line i, then we add U'_i to the velocity $u_i(y)$ and the average pressure correction P'_i to all the pressures at the grid nodes located downstream of this line in order to maintain the flux of fluid Q; see Figure 1(a), where

$$P_i = P^*_i + \alpha_p P'_i. \tag{6}$$

If we apply the average pressure correction to the lines from i to $i+k$, then the pressure P^*_i on every line is corrected by the total pressure corrections upstream, namely (see Figure 1(b))

$$P_i = P^*_i + \alpha_p P'_i, \tag{7}$$

$$P_{i+1} = P^*_{i+1} + \alpha_p(P'_i + P'_{i+1}), \tag{8}$$

$$P_{i+k} = P^*_{i+k} + \alpha_p(P'_i + P'_{i+1} + \ldots + P'_{i+k}), \tag{9}$$

and the pressure at all the grid nodes located downstream of the line k are corrected by

$$\alpha_p(P'_i + P'_{i+1} + \ldots + P'_{i+k}), \tag{10}$$

where α_p is the relaxation factor.

3. DISCUSSION OF THE TEST CALCULATIONS

In order to illustrate the effect of the average correction technique on the convergence of the iterative procedure, calculations have been performed for the laminar flow in a plane, two-dimensional sudden expansion using staggered control volumes at a Reynolds number ($Re = \rho U_0 h/\mu$) of 100 and $H/h = 2$, where U_0 is the uniform fluid velocity upstream of the expansion, H is the half-width of the downstream channel, h is the half-width of the upstream channel, ρ is the density of the fluid and μ is the viscosity of the fluid. Thus the flow is symmetrical about the midplane of the channel and the solution domain is as shown in Figure 2. On the walls of the channel the no-slip condition has been imposed and on the line of symmetry the gradients of all variables in the y-direction were set to zero. At the channel exit the gradients of all variables in the x-direction were set to zero. Both the first-order upwind and QUICK[3] schemes have been used for the discretization of the convection term in the momentum equations, uniform grid systems with an aspect ratio of the grid sizes Δx to Δy of 2·5 for mesh numbers 70 × 20, 140 × 40 up to a very fine grid of 210 × 60 were used to cover the solution domain with $-2{\cdot}5h \leqslant x \leqslant 15h$ and $0 \leqslant y \leqslant 2h$ and the SIMPLEC algorithm[4] was used in all the present calculations. Larger solution domains have been investigated, but for the present

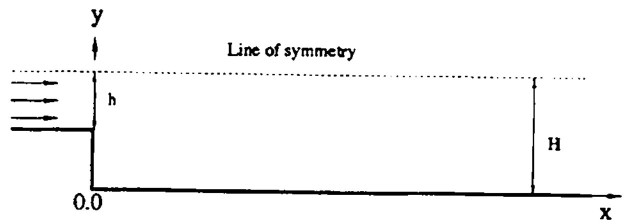

Figure 2. Geometry of solution domain for plane, two-dimensional sudden expansion

value of the Reynolds number the range of values of x considered was found to be sufficiently large to obtain an accurate solution. The Thomas tridiagonal matrix algorithm (TDMA) in a line-by-line manner, which alternates in direction, was used to solve the discretized form of the momentum equation. The pressure correction equation was solved by both the line-by-line TDMA and the strongly implicit procedure (SIP) of Stone[5] with the cancellation parameter $\alpha = 0{\cdot}8$, which produces a fast rate of convergence. For the momentum and pressure correction equations, one sweep and six sweeps respectively were performed when using the TDMA. When using the SIP, the convergence criterion for terminating the iterative procedure was based on the value of the ratio of the sum of the absolute residuals in the pressure correction equation after each iteration to its initial value. When this ratio is less than 0·1 or the number of iterations reaches six, then the inner iterative procedure is terminated. In order to compare the rates of convergence, for convenience, the sum, normalized by the inlet mass flux Q, of the mass residuals over all the control volumes was used as the measure of convergence, although other measurements for convergence are available. In order to check the possibility of producing multiple solutions by the average correction technique, two initial guessed values for the fluid velocities, namely (a) $u = 0$, $v = 0$ and (b) $u = 1$, $v = 0$ everywhere, were used and we observed that for each grid system all the numerical results produced by implementation of the average correction are almost identical with those without the average correction technique when the mass residue is less than 10^{-5}. Thus the average correction technique does not appear to produce multiple solutions. In all further discussions in this paper the initial guessed values for the unknown fluid velocity and pressure are set to be identically zero.

For the case where the relaxation factor in the momentum equation and pressure correction equation are taken as $\alpha_u = 0{\cdot}7$ and $\alpha_p = 0{\cdot}8$, Figure 3 shows the convergence histories of the mass residual on the 70×20 grid when the upwind scheme is used. It is observed that the TDMA produces almost the same rate of convergence as the SIP, but the former produces a stronger oscillation of the variation in the mass residual than the latter. When only the average velocity correction is implemented in the TDMA, the mass residual reduces rapidly at the beginning of the iteration procedure. However, this rate of convergence slows down after about 150 iterations and the rate of convergence becomes much slower after about 200 iterations. The reason for this is that the average velocity correction can quickly produce a velocity distribution which satisfies global mass conservation, but at the same time the mass residual on every control volume is also reduced. This reduced mass residual can only produce a small pressure correction and this in turn requires more iterations in order to build up the pressure distribution. Therefore we conclude that if only the average velocity correction is implemented in the TDMA, the rate of convergence becomes slower than for the TDMA without the average velocity correction. The phenomenon presented in Figure 3 is also found to occur when using the QUICK scheme and finer grids and therefore these results are not shown.

We also observe in Figure 3 that when only the average pressure correction is employed in both the TDMA and the SIP, the rate of convergence is slightly improved and there are no oscillations in the convergence history. However, we have found that when the initial guessed average velocity is much

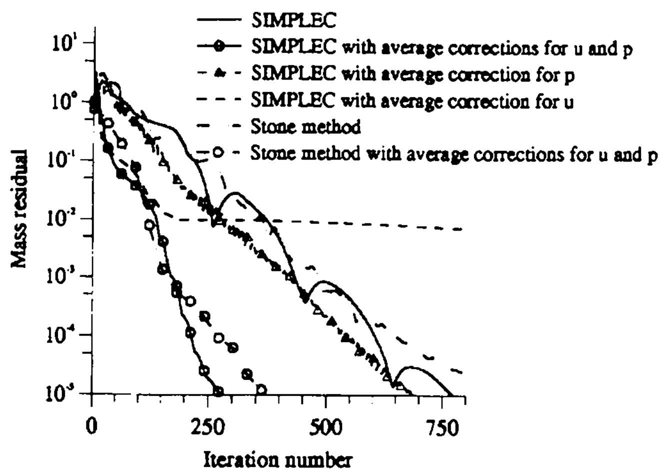

Figure 3. Convergence histories of mass residual on 70 × 20 grid when using upwind scheme

larger than and in the opposite direction to the solution average velocity, the iterative procedure may diverge if only the pressure correction technique is employed. For example, in Figure 1(a), suppose that the correct average velocity is such that $U_i > 0$, i.e. the fluid flows in the x-direction, and the updated average velocity is such that $U_i^* < 0$, i.e. the updated fluid velocity is in the negative x-direction; then from equation (2) we have $U_i' > 0$. Therefore we would expect to produce an average pressure correction such that $P_i' < 0$ in order to make the pressure downstream smaller so that the fluid can flow in the positive x-direction. However, if $|U_i^*| > |U_i|$, equation (3) results in $P_i' > 0$, namely a larger pressure exists downstream, and this is clearly unreasonable. Under such circumstances the use of only the average pressure leads to divergence in the iterative procedure. However, the average fluid flow in the negative x-direction may be avoided by the use of the average velocity correction, because the average velocity correction keeps the updated average velocity always close to the true average value. Therefore the use of both average velocity and pressure corrections produced by equation (3) may yield a much better and faster rate of convergence. In Figure 3 we observe that when using both average velocity and pressure corrections, the rate of convergence is significantly improved. We have also used equation (5) to produce the average pressure correction and observed that equation (5) does not have the divergence problem of equation (3), because the use of the real average velocity does not produce unreasonable $P_i' > 0$ when $U_i^* < 0$. Similar convergence histories of the mass residual occur for the other two grid sizes.

Figure 4 shows the effect of the relaxation factors α_u and α_p on the rate of convergence for the 70 × 20, 140 × 40 and 210 × 60 grid systems when the QUICK scheme was used and the convergence criterion on the mass residual was 10^{-5}. It is observed that the smallest number of iterations required in the SIMPLEC algorithm and the SIP algorithm with the average corrections is about 40 per cent of that required in the SIMPLEC algorithm and the SIP algorithm in all the coarse and fine grid situations. As the grid is refined, the number of iterations required for convergence increases. However, when both average velocity and pressure corrections are used, the number of iterations required for convergence when employing the 210 × 60 grid system is the same as that for the 70 × 20 grid when only the SIMPLEC algorithm is employed. Figure 4 shows that when the average correction technique is employed, the range of relaxation factors for which convergence is possible is much larger than when using only the SIMPLEC algorithm and the SIP algorithm. As the grid is refined, we observe that the range of possible relaxation factors becomes narrower when using the SIMPLEC algorithm and the SIP algorithm, but when the average correction technique is employed, this range is large and almost independent of the refinement of the grid. We also find that the rate of convergence of the iterative procedure is mainly determined by the choice of the value of α_u and that the value of α_p has no significant effect on the rate of convergence; for example, there is little difference in the number of iterations required for $\alpha_p = 0{\cdot}5$ and $0{\cdot}8$ and this is also true for all the grid systems. Although all the above conclusions have been reached using the convergence criterion that the mass residual be less than 10^{-5}, similar observations can be made for other convergence criteria.

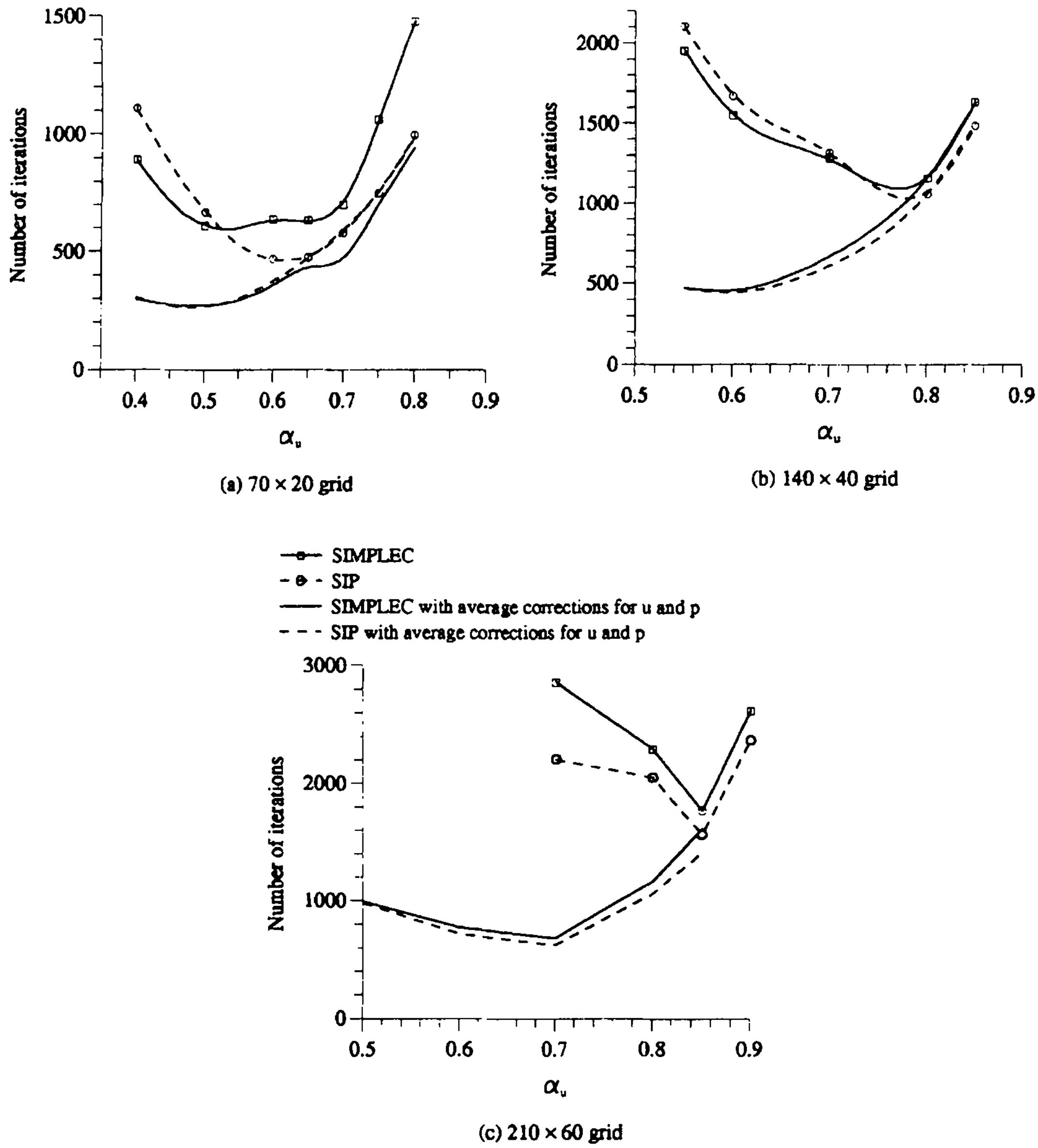

Figure 4. Effects of α_u and grid size on rate of convergence

The average velocity and pressure corrections have also been used to calculate flows in cavities and the complex three-dimensional laminar and turbulent flows around samplers at various angles to the oncoming flow.[6] It has been found that all the general observations made for the problem investigated in detail in this paper, namely the plane, two-dimensional sudden expansion flow, are valid. It is found in the three-dimensional sampling problem that if the average velocity and pressure corrections are not implemented, it is impossible to obtain accurate, convergent results.[6]

4. CONCLUSIONS

The main conclusion of this investigation is that although the average velocity correction assists in avoiding divergence of the iterative procedure, it slows down the rate of convergence. However, the average pressure correction can improve the rate of convergence, but the initial choice for the fluid velocity affects this rate of convergence. Therefore, on using both average velocity and pressure corrections, one can achieve a better and faster rate of convergence.

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